Using Sound Wave Measurements to Measure Angles
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Overview

In this experiment, we will make measurements of the phase and frequency of sound waves from a sound source using two different microphones. We will then use the frequency and the spacing between the microphones to determine the angles of the triangle formed by the microphones and the sound source.

Introductory Science Concepts

Some telescopes, particularly radio and submillimeter telescopes, are designed to use multiple mirrors or antennae to observe objects. Examples of such telescopes include the Very Large Array in New Mexico and the Atacama Large Millimeter/submillimeter Telescope in Chile (see Figure 1).

Figure 1: The Atacama Large Millimeter/submillimeter Array in Chile. The individual antennae work together as a single telescope. Credit: ALMA (ESO/NAOJ/NRAO), C. Padilla.

When the telescope observes an object, all antennae observe the same frequency of electromagnetic radiation from the object. However, the antennae will be at different distances from the object, so the electromagnetic waves will be in different phases, as seen in Figure 2.

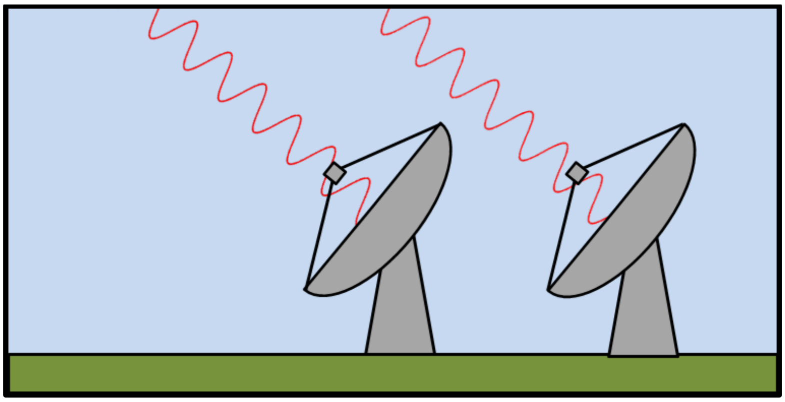

Figure 2: A diagram showing two radio antennae observing radio waves from a single source. The radio wave travels a longer distance to reach the antennae on the left and is observed in a different phase. In this case, the observed waves are about 90° out of phase.

Each antenna transforms the electromagnetic wave into an electronic signal. The signals from pairs of antennae are then added together, which is called interference. If the two signals are close to perfectly in phase, the two signals add together to form a larger wave, which is called constructive interference. If the two signals are close to 180° out of phase, then the two signals cancel each other out when they are added together, which is called destructive interference. This can be seen in Figure 3.

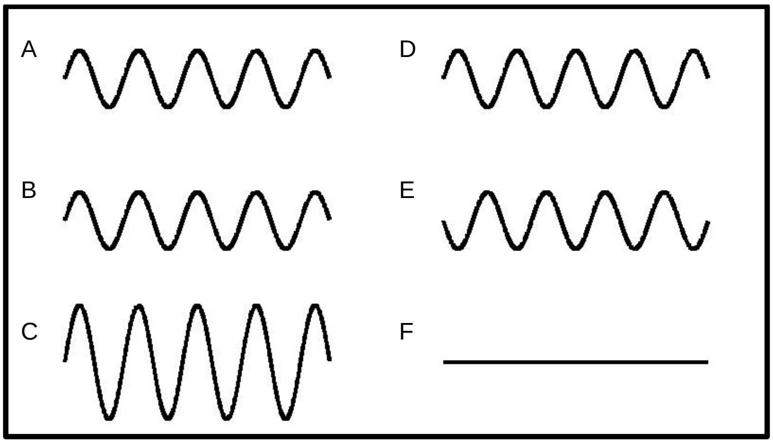

Figure 3: A diagram showing constructive and destructive interference. On the left are two sine waves A and B that are in phase. When they are added together, they produce a sine wave C that has an amplitude twice as large. On the left are two sine waves D and E that are 180° out of phase. When they are added together, the waves cancel out, producing no wave as seen in F.

In astronomy, the spacing of the antennae and the exact position of the source in the sky will strongly affect the interference pattern made from the signals from two antennae. Small changes in the source position could cause the waves to constructively or destructively interfere. As the Earth turns, astronomical objects will move across the sky, and the interference pattern will change.

Science Concepts in This Experiment

In this experiment, we will use concepts related to interference to measure distances and angles. Instead of measuring light waves from a source, we will be measuring sound waves. We will put two microphones at an unknown distance from a sound source. At first, we will put the two microphones right next to each other. We will then move one of them in a direction perpendicular to the direction to the sound source, as shown in Figure 4. As we do this, the observed sound signals will move out of phase. When the signals are back in phase, we can then measure the frequency and the distance between the microphones to determine the distance to the source. Following this, we can measure the angles in the triangle formed by the sound source and the two microphones. (In astronomy, these techniques are not used to measure distances because the objects are very far away, but they are used to measure precise angles.)

We will use the equation

$$v = \lambda f \qquad (1)$$

where v is the speed of sound, λ is the wavelength of the sound, and f is the frequency of the sound. We can write the distance from the sound source to microphone 1 as a number of wavelengths n, or

$$d_1 = n\lambda \qquad (2)$$

(The number n may not be a round number.) When the waves are constructively interfering, the distance to microphone 2 will be 1 wavelength further away from the sound source, or

$$d_2 = (n+1)\lambda \qquad (3)$$

Measuring the wavelength of sound is difficult. It is usually easier to measure the frequency of sound, use 343 m/s as the speed of sound (at 20° C and at sea level), and then apply Equation 1 to calculate the wavelength. This changes Equations 2 and 3 into

$$d_1 = nv/f \qquad (4)$$

$$d_2 = (n+1)v/f \qquad (5)$$

We still do not know how many wavelengths are between the source and the two microphones. However, if we measured the distance between the microphones ($d_\perp$), we can use Pythagoras's theorem to relate the other distances to this measured distance:

$$d_2^2 = d_1^2+d_\perp^2 \qquad (6)$$

Substituting Equations 4 and 5 into Equation 6 gives

$$((n+1)v/f)^2 = (nv/f)^2+d_\perp^2 \qquad (7)$$

We can then rearrange this as

$$((n+1)^2-n^2)v^2/f^2 = d_\perp^2 \qquad (8)$$

$$(2n+1)v^2/f^2 = d_\perp^2 \qquad (9)$$

$$n = (d_\perp^2 f^2/v^2-1)/2 \qquad (10)$$

We can then use Equation 10 as well as measurements of $d_\perp$ and f to determine the number of wavelengths n between microphone 1 and the sound source. The two acute angles in the right triangle can be calculated using

$$\Theta_{12} = \cos^{-1}(d_1/d_2) \qquad (11)$$

$$\theta_{2\perp} = \sin^{-1}(d_1/d_2) \qquad (12)$$

Because astronomical objects are so far away, these techniques cannot be used to find the distances to sources. However, these techniques are simplified version of the techniques astronomers use to find the positions of astronomical objects in the sky using interferometry.

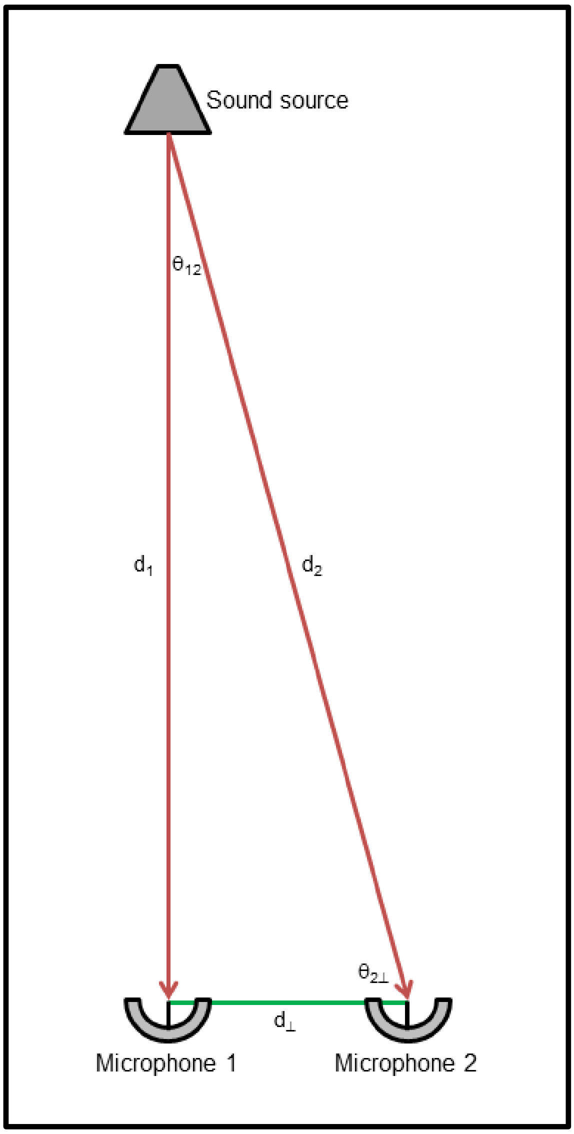

Figure 4: The experimental set-up for this project. The experiment consists of a sound source producing a constant tone and two microphones (shown here as parabolic microphones) observing the sound. The distances the sound wave travels from the sound source to each microphone are shown as red arrows and are labelled as d_1 and d_2. The distance between microphones is shown with a green line and is labelled $d_\perp$.

Equipment

- *Sound source* – You can use a computer, a smartphone, or other tone or signal generator plugged into a speaker. It is possible to download tone generator apps for Android and Apple devices; we use Tone Generator Pro on an iPod Touch.
- *Two high-sensitivity microphones* – These should be highly amplified microphones, not standard microphones. We use two Sonic Sleuth microphones, but these and similar microphones are also sold under other names (such as Bionic Ear and Sonic Explorer).
- *Two stereo audio cables*
- *Two stereo input to mono output cables*
- *Stereo input to two mono output splitter*
- *Computer with oscilloscope program* – We are using Soundcard Oscilloscope by Christian Zeitnitz (http://www.zeitnitz.de/Christian/scope_en) on a netbook running Windows 8. This software is free for public education use. Other software, including software for Mac and Linux, is available on the web. You will need a program that can plot the signal from the right and left channels separately.

Procedure

1. Turn on the computer and start the oscilloscope program.

2. Plug the two stereo input to mono output converters into the splitter. Plug the splitter into the microphone input on your computer. Use the audio cables to plug each microphone into each converter.

3. Set up the sound source so that it produces a single tone between 200 and 800 Hz. (It is possible to do this experiment with lower tones, but the spacing between microphones may end up very large. Higher frequencies become too irritating to listen to, and the spacing between microphones will probably be too small.) If you are using stereo speakers, be certain that only one speaker is on, or else the sound from the two speakers will undergo constructive and destructive interference.

4. If your oscilloscope software has frequency filtering options, we recommend using this to filter out other frequencies. Selecting frequencies between 50 Hz above and 50 Hz below the frequency from your sound source will work well.

5. Set up both microphones at a distance of a few metres from the sound source. (You can measure the exact distance now or at the end of the experiment.) Use the microphones to measure the signal from the sound source. You should be measuring two sound waves that are in phase with each other. If your microphones have volume controls, adjust the volume controls so that the two signals match.

6. Try one of the following options:
    a. (Easier version, but less like interferometric telescopes.) Move microphone 2 perpendicular to the direction to the sound source. The phase of one sound wave will shift relative to the other. When the sound waves are in phase again, measure the distance between the microphones.
    b. (Harder version, but more like real interferometric telescopes.) Set up the oscilloscope so that it adds the signal from the two oscilloscopes together. The sound waves will constructively interfere. Move microphone 2 perpendicular to the direction to the sound source. The sound waves will shift out phase relative to each

other. They will first destructively interfere and then constructively interfere again. When the interference pattern reaches a peak amplitude, measure the distance between the two microphones.

7. Use Equation 10 to calculate the number of wavelengths between the microphones and the sound source. (It may be a decimal number and not a round integer.) Then use Equation 2 to calculate the distance in metres. Compare this to the actual distance between the microphones (which you can measure now if you have not already done so), and then report the two distances.
8. Use Equation 11 and 12 to calculate the sizes of the angles in the triangle formed by the sound source and the two microphones. Compare this to what you obtain using a protractor to measure the angle that microphone 2 was turned to so that it pointed at the source.

Additional Exercises

1. Try using both methods described in step 6 of the Procedure section. See which one produces more accurate results.
2. Try repeating the experiment with a different frequency (one that is 2 times higher or lower in frequency). Do you get the same answer?
3. Instead of moving microphone 2 to the location where the signals are in phase, try moving microphone 2 to the location where the signals are out of phase (or where the added signals from the two microphones reaches a minimum amplitude or just becomes noise). Instead of using Equation 10, use

$$n = d_{\perp}^2 f^2/v^2 - 0.25 \qquad (13)$$

to calculate n. Does this give the same result as when you positioned the microphones so that their signals were in phase?